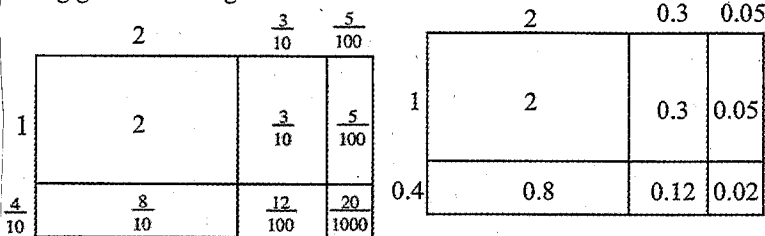# MULTIPLYING DECIMALS

There are at least two ways to multiply decimals. One way is to convert the decimals to fractions and use your knowledge of fraction multiplication to compute the answer. The other way is to use the method that you have used to multiply integers; the only difference is that you need to keep track of where the decimal point is (place value) as you record each line of your work.

The examples below show how to compute $1.4(2.35)$ both ways by using generic rectangles.

	2	$\frac{3}{10}$	$\frac{5}{100}$
1	2	$\frac{3}{10}$	$\frac{5}{100}$
$\frac{4}{10}$	$\frac{8}{10}$	$\frac{12}{100}$	$\frac{20}{1000}$

	2	0.3	0.05
1	2	0.3	0.05
0.4	0.8	0.12	0.02

If you carried out the computation as shown above, you can calculate the product in either of the two ways shown at right. In the first one, you write down all of the values in the smaller rectangles within the generic rectangle and add the six numbers. In the second example, you combine the values in each row and then add the two rows. You usually write the answer as 3.29 since there are zero thousandths in the product.

$$
\begin{array}{r}
2.35 \\
\times \ 1.4 \\
\hline
0.020 \\
0.12 \\
0.8 \\
0.05 \\
0.3 \\
2.0 \\
\hline
3.290
\end{array}
$$

$$
\begin{array}{r}
2.35 \\
\times \ 1.4 \\
\hline
0.940 \\
2.35 \\
\hline
3.29
\end{array}
$$

ORDER OF OPERATIONS

Mathematicians have agreed on an **order of operations** for simplifying expressions.

Original expression:

$$(10 - 3 \cdot 2) \cdot 2^2 - \frac{13 - 3^2}{2} + 6$$

Circle expressions that are grouped within parentheses or by a fraction bar:

$$\boxed{(10 - 3 \cdot 2)} \cdot 2^2 - \frac{\boxed{13 - 3^2}}{2} + 6$$

Simplify *within* circled expressions using the order of operations:

- Evaluate exponents.

$$\boxed{(10 - 3 \cdot 2)} \cdot 2^2 - \frac{\boxed{13 - 3 \cdot 3}}{2} + 6$$

- Multiply and divide from left to right.

$$\boxed{(10 - 6)} \cdot 2^2 - \frac{\boxed{13 - 9}}{2} + 6$$

- Combine terms by adding and subtracting from left to right.

$$(4) \cdot 2^2 - \frac{4}{2} + 6$$

Circle the remaining terms:

$$\boxed{(4) \cdot 2^2} - \boxed{\frac{4}{2}} + \boxed{6}$$

Simplify *within* circled terms using the order of operations as above:

$$4 \cdot 2 \cdot 2 - \frac{4}{2} + 6$$

$$16 - 2 + 6$$

$$20$$

ADDING AND SUBTRACTING MIXED NUMBERS

To **add or subtract mixed numbers**, you can either add or subtract their parts, or you can change the mixed numbers into fractions greater than one.

To add or subtract mixed numbers by adding or subtracting their parts, add or subtract the whole-number parts and the fraction parts separately. Adjust if the fraction in the answer would be greater than one or less than zero. For example, the sum of $3\frac{4}{5} + 1\frac{2}{3}$ is calculated at right above.

$$
\begin{aligned}
3\frac{4}{5} &= 3 + \frac{4}{5} \cdot \frac{3}{3} = 3\frac{12}{15} \\
+1\frac{2}{3} &= 1 + \frac{2}{3} \cdot \frac{5}{5} = +1\frac{10}{15} \\
\hline
&\qquad\qquad 4\frac{22}{15} = 5\frac{7}{15}
\end{aligned}
$$

It is also possible to add or subtract mixed numbers by first changing them into fractions greater than one. Then add or subtract in the same way you would if they were fractions between 0 and 1. For example, the sum of $2\frac{1}{6} + 1\frac{4}{5}$ is calculated at right.

$$
\begin{aligned}
2\frac{1}{6} + 1\frac{4}{5} &= \frac{13}{6} + \frac{9}{5} \\
&= \frac{13}{6} \cdot \frac{5}{5} + \frac{9}{5} \cdot \frac{6}{6} \\
&= \frac{65}{30} + \frac{54}{30} \\
&= \frac{119}{30} \\
&= 3\frac{29}{30}
\end{aligned}
$$

MULTIPLYING FRACTIONS

You can find the product of two fractions, such as $\frac{2}{3}$ and $\frac{3}{4}$, by multiplying the numerators (tops) of the fractions together and dividing that by the product of the denominators (bottoms). So $\frac{2}{3} \cdot \frac{3}{4} = \frac{6}{12}$, which is equivalent to $\frac{1}{2}$. Similarly, $\frac{4}{7} \cdot \frac{3}{5} = \frac{12}{35}$. If you write this method in algebraic terms, you would say $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$.

The reason that this rule works can be seen using an area model of multiplication, as shown at right, which represents $\frac{2}{3} \cdot \frac{3}{4}$. The product of the denominators is the total number of smaller rectangles, while the product of the numerators is the number of the rectangles that are double-shaded.
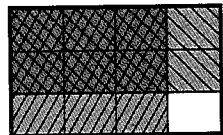
MULTIPLYING MIXED NUMBERS

An efficient method for **multiplying mixed numbers** is to convert them to fractions greater than one, find the product as you would with fractions less than one, and then convert them back to a mixed number, if necessary. (Note that you may also use generic rectangles to find these products.) Here are three examples:

$$1\frac{2}{3} \cdot 2\frac{3}{4} = \frac{5}{3} \cdot \frac{11}{4} = \frac{55}{12} = 4\frac{7}{12}$$

$$1\frac{3}{5} \cdot \frac{2}{9} = \frac{8}{5} \cdot \frac{2}{9} = \frac{16}{45}$$

$$2\frac{1}{3} \cdot 4\frac{1}{2} = \frac{7}{3} \cdot \frac{9}{2} = \frac{63}{6} = 10\frac{3}{6} = 10\frac{1}{2}$$

RATIOS

A **ratio** is a comparison of two numbers, often written as a quotient; that is, the first number is divided by the second number (but not zero). A ratio can be written in words, in fraction form, or with colon notation. Most often, in this class, you will either write ratios in the form of fractions or state the ratios in words.

For example, if there are 38 students in a school band and 16 of them are boys, you can write the ratio of the number of boys to the number of girls as:
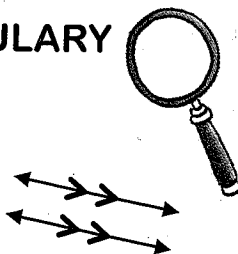
16 boys to 22 girls $\frac{16 \text{ boys}}{22 \text{ girls}}$ 16 boys : 22 girls

PARALLELOGRAM VOCABULARY

Two lines in a plane (a flat surface) are **parallel** if they never meet no matter how far they extend. The distance between the parallel lines is always the same. The marks " >> " indicate that the two lines are parallel.

The **distance** between two parallel lines or segments is the length of a line segment that is **perpendicular** (its ends form right angles) to both parallel lines or segments. In the diagram at right, the height (h) is the distance between the two parallel lines. It is also called the **perpendicular distance**.
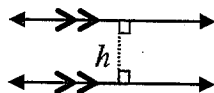
A **parallelogram** is a quadrilateral (a four-sided figure) with two pairs of parallel sides. Any side of a parallelogram can be used as a base. The height (h) is perpendicular to one of the pairs of parallel bases (b), or an extension of a base like the dashed line in the example at lower right.
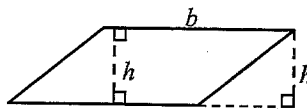
AREA OF A TRIANGLE

Since two copies of the same triangle can be put together along a common side to form a parallelogram with the same base and height as the triangle, then the **area of a triangle** must equal half the area of the parallelogram with the same base and height.

Therefore, if b is the base of a triangle and h is the height of the triangle, you can think of triangles as "half parallelograms" and calculate the area of any triangle:

$$A = \tfrac{1}{2} b \cdot h$$
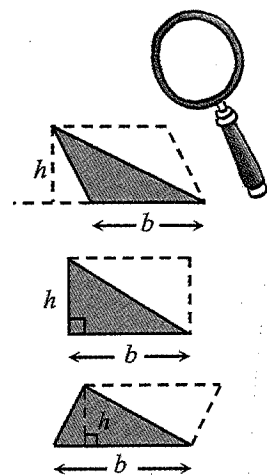
BASE AND HEIGHT OF A RECTANGLE

Any side of a rectangle can be chosen as its **base**. Then the **height** is either of the two sides that intersect (meet) the base at one of its endpoints. Note that the height may also be any segment that is **perpendicular** to (each end forms a right angle (90°) with) both the base and the side opposite (across from) the base.

In the first rectangle at right, side $\overline{BC}$ is labeled as the base. Either side, AB or DC, is a height, as is segment FE.
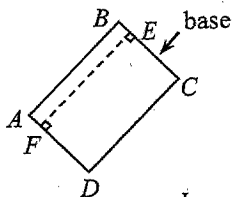
In the second rectangle, side $\overline{GJ}$ is labeled as the base. Either side, HG or IJ, is a height, as is segment MN. Segment GL is not a height, because it is not perpendicular to side GJ.
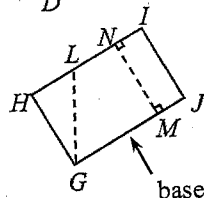
AREA OF A TRAPEZOID

There are multiple ways to divide a trapezoid and rearrange the pieces into a parallelogram with the same area. For example, the trapezoid can be divided parallel to its two bases to create two smaller trapezoids that are each half of the height of the original trapezoid. Those two pieces can be rearranged into a parallelogram, as shown below.
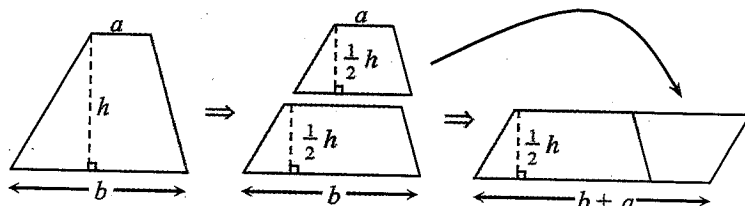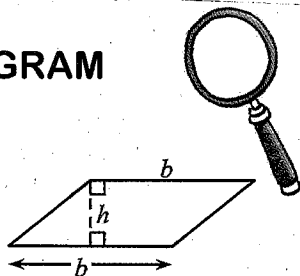
Therefore, to find the **area of a trapezoid**, find the product of half of the height (h) and the sum of the two bases (a and b).
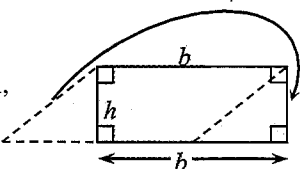
$$A = \tfrac{1}{2} h(a+b)$$

AREA OF A PARALLELOGRAM

A parallelogram can be rearranged into a rectangle with the same base length and height. Since the area of a shape does not change when it is cut apart and its pieces are put together in a different arrangement (a principle called **conservation of area**), the area of the parallelogram must equal the product of its base and height.

Therefore, to find the area of a parallelogram, find the product of the length of the base (b) and the height (h).

$$A = b \cdot h$$